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Chiral supersymmetric Gepner model orientifolds

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Abstract: We explicitly construct A-type orientifolds of supersymmetric Gepner models. In order to reduce the tadpole cancellation conditions to a treatable number we explicitly work out the generic form of the one-loop Klein bottle, annulus and Möbius strip amplitudes for simple current extensions of Gepner models. Equipped with these formulas, we discuss two examples in detail to provide evidence that in this setting certain features of the MSSM like unitary gauge groups with large enough rank, chirality and family replication can be achieved.

Keywords: D-branes, Superstring Vacua, Conformal Field Models in String Theory.

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1. Introduction

Various ways for constructing phenomenologically semi-realistic four-dimensional string vacua have been followed in the past. After focusing solely on the heterotic string, the realization that D-branes play a very crucial role in string theory opened up the possibility to look out for new string vacua where these D-branes are actually present. The right framework to study such models are orientifold models, where the presence of the D-branes is implied by tadpole cancellation conditions. These orientifold models were the subject of intense study during the last years and it became clear that there are essentially two approaches to get chiral semi-realistic models. One can either get chiral fermions from D-branes on orbifold singularities [1]–[4] or from intersecting D-branes [5]–[10]. Both approaches have been followed extensively and many semi-realistic models have been proposed so far, but it is certainly fair so say that none of them gives indeed rise to physics of the Standard Model, but at least some of its rough data could be realized such as the right gauge group and chiral fermions in three generations.

So far these constructions were mostly limited to toroidal orbifold backgrounds, which, of course, cover only a very small subset of all possible closed string backgrounds. It is therefore desirable to also study orientifold models on more general non-flat Calabi-Yau spaces [11]. One way to approach this problem is to start with a class of exactly solvable superconformal field theories which are known to describe certain points deep inside the Kähler moduli space of Calabi-Yau manifolds. These models are commonly known as Gepner models [12, 13]. The construction of boundary [14]–[20] and crosscap states [21]–[32] in these models has been under investigation during the last years. However, there have been only quite a few attempts so far to really construct fully fledged orientifolds of Gepner models [33]–[37]. Whereas in the beginning the construction of these orientifolds was carried out in a case by case study, only very recently [37] it was possible to derive (at least for all levels being odd) quite generic formulas for all one-loop partition functions including the highly non-trivial Möbius strip amplitude containing important sign factors. Moreover, amazingly simple expressions for the tadpole cancellation conditions were derived. We regard these general equations as a useful starting point for an explicit and general study of such models. One has to distinguish two different kinds of orientifolds which are called A-type and B-type. It turned out that for B-type models the number of tadpole cancellation conditions is much smaller than for the A-type models and therefore much easier to solve.

This latter point was the main reason why in [37] the primary focus was on B-type models. Even though the tadpole cancellation conditions could easily be solved, the prize one had to pay was that due to general arguments these B-type models were always non-chiral. Therefore from the phenomenological point of view they are not very interesting. Additionally, from the technical point of view it would have been desirable to confirm that the signs in the Möbius strip amplitude indeed lead to anomaly cancellation of the effective low energy theory. But, of course, without chiral fermions such a check could not be made.

In this paper we continue the study of orientifolds of Gepner models focusing on the A-type models, for which one expects that chiral models should be possible. As we mentioned already, the main obstacle is the huge number of tadpole cancellation conditions (of the order 10^2), which made it hard to determine any other solution than the generic one with gauge group SO(4) or SP(4). In order to reduce this number we consider simple current extensions [38] of Gepner models and generalize the explicit results for the one-loop amplitudes of [37] to this case. Note that generic expressions for boundary and crosscap states for simple current extensions were presented in [25], which could also serve as an alternative starting point for such a construction. For the set of simple currents of Gepner models with all levels being odd we derive the generic form of the Klein bottle, annulus and Möbius strip amplitudes. Equipped with these formulas, we study two models in great detail and show that indeed certain aspects of the Standard Model, including chirality, can be realized in this setting. The aim of this paper is not to provide a systematic search for the MSSM in this class of models, but merely to provide the relevant formalism and to give evidence that such a search might be worthwhile carrying out.

This paper is organized as follows. In section 2, we review some facts about Gepner models. In section 3, after some comments about the simple current extension of modular invariants for general CFTs, we apply these methods to the diagonal and charge conjugate partition function of the Gepner model. Section 4 contains the derivation of the corresponding orientifolds in terms of the computation of the simple current extended Klein bottle amplitude of A-type. In principle due to the Greene-Plesser construction [39] of the mirror, A-type models would be sufficient, but nevertheless we provide explicit formulas for the B-type Klein bottle amplitude in appendix A. In section 5, we review some of the important aspects of RS-boundary states including the loop and tree channel annulus amplitudes. The derivation of the Möbius strip amplitudes is the subject of section 6, where for simplicity we restrict ourselves to the explicit computation of the NS sector amplitudes. In section 7, we present the general tadpole cancellation conditions, followed by a general analysis of the massless open string sector including the gauge sector. The techniques developed are employed to discuss a couple of examples in section 8. Finally, section 9 contains our conclusions.

Note: while this work was in its very final stage we became aware of the paper Brunner et. al. [40], which has some overlap with our work.

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2. Review of Gepner models

Let us briefly review some aspects of Gepner models needed in the following. In light-cone gauge, the internal sector of a type-II compactification to four dimensions with N=2 supersymmetry is given by tensor products of the rational models of the N=2 super Virasoro algebra with total central charge $c = 9$ [12, 13]. Space-time supersymmetry is achieved by a GSO projection, which can be described by a certain simple current in the superconformal field theory.

The minimal models are parametrized by the level $k = 1, 2, \ldots$ and have central charge

$$c = \frac{3k}{k+2}\,. \tag{2.1}$$

Since $c < 3$, one achieves the required value $c = 9$ by using tensor products of such minimal models $\bigotimes_{j=1}^{r}(k_j)$. The finite number of irreducible representations of the N=2 Virasoro algebra of each unitary model are labeled by the three integers (l, m, s) in the range

$$l = 0, \dots k\,, \qquad m = -k-1, -k, \dots k+2\,, \qquad s = -1, 0, 1, 2 \tag{2.2}$$

with $l + m + s = 0 \bmod 2$. Actually, the identification between (l, m, s) and $(k - l, m + k + 2, s + 2)$ reveals that the range (2.2) is a double covering of the allowed representations. The conformal dimension and charge of the highest weight state with label (l, m, s) is given by

$$\begin{aligned} \Delta^l_{m,s} &= \frac{l(l+2) - m^2}{4(k+2)} + \frac{s^2}{8} \\ q^l_{m,s} &= \frac{m}{(k+2)} - \frac{s}{2}\,. \end{aligned} \tag{2.3}$$

Note that these formulas are only correct modulo one and two respectively. To obtain the precise conformal dimension h and charge from (2.3) one first shifts the labels into the standard range $|m - s| \le l$ by using the shift symmetries $m \to m + 2k + 4$,$s \to s + 4$ and the reflection symmetry. The NS-sector consists of those representations with even s, while the ones with odd s make up to the R-sector.

In addition to the internal N=2 sector, one has the contributions with $c = 3$ from the two uncompactified directions. The two world-sheet fermions $\psi^{2,3}$ generate a $\mathrm{U}(1)_2$ model whose four irreducible representations are labeled by $s_0 = -1, \dots 2$ with highest weight and charge modulo one and two respectively

$$\Delta_{s_0} = \frac{s_0^2}{8}\,, \qquad q_{s_0} = -\frac{s_0}{2}\,. \tag{2.4}$$

The GSO projection means in the Gepner case that one projects onto states with odd overall U(1) charge $Q_{\mathrm{tot}} = q_{s_0} + \sum_{j=1}^{r} q^{l_j}_{m_j,s_j}$. Moreover, to have a good space-time interpretation one has to ensure that in the tensor product only states from the NS respectively the R sectors couple among themselves.

These projections are described most conveniently in the following notation. First one defines some multi-labels

$$\lambda = (l_1, \dots, l_r)\,, \qquad \mu = (s_0; m_1, \dots m_r; s_1, \dots, s_r) \tag{2.5}$$

and the respective characters

$$\chi^{\lambda}_{\mu}(q) = \chi_{s_0}(q)\, \chi^{l_1}_{m_1,s_1}(q), \dots, \chi^{l_r}_{m_r,s_r}(q)\,. \tag{2.6}$$

In terms of the vectors

$$\beta_0 = (1; 1, \dots, 1; 1, \dots, 1)\,, \qquad \beta_j = (2; 0, \dots, 0; 0, \dots, 0, \underbrace{2}_{j^{\mathrm{th}}}, 0, \dots, 0) \tag{2.7}$$

and the following product

$$\begin{aligned} Q_{\mathrm{tot}} &= 2\beta_0 \bullet \mu = -\frac{s_0}{2} - \sum_{j=1}^{r} \frac{s_j}{2} + \sum_{j=1}^{r} \frac{m_j}{k_j + 2}\,, \\ \beta_j \bullet \mu &= -\frac{s_0}{2} - \frac{s_j}{2}\,, \end{aligned} \tag{2.8}$$

the projections one has to implement are simply $Q_{\rm tot} = 2\beta_0 \bullet \mu \in 2\mathbb{Z}+1$ and $\beta_j \bullet \mu \in \mathbb{Z}$ for all $j = 1, \ldots, r$. Gepner has shown that the following GSO projected partition function

$$Z_D(\tau,\overline{\tau}) = \frac{1}{2^r}\frac{({\rm Im}\tau)^{-2}}{|\eta(q)|^4}\sum_{b_0=0}^{K-1}\sum_{b_1,\ldots,b_r=0}^{1}\sum_{\lambda,\mu}^{\beta}(-1)^{s_0}\;\chi^{\lambda}_{\mu}(q)\,\chi^{\lambda}_{\mu+b_0\beta_0+b_1\beta_1+\cdots+b_r\beta_r}(\overline{q}) \tag{2.9}$$

is indeed modular invariant and vanishes due to space-time supersymmetry. Here $K = {\rm lcm}(4, 2k_j+4)$ and $\sum^{\beta}$ means that the sum is restricted to those λ and μ in the range (2.2) satisfying $2\beta_0 \bullet \mu \in 2\mathbb{Z}+1$ and $\beta_j \bullet \mu \in \mathbb{Z}$. The factor 2^r due to the field identifications guarantees the correct normalization of the amplitude. In the partition function (2.9) states with odd charge are arranged in orbits under the action of the β vectors. Therefore, although the partition function is non-diagonal in the original characters, for all levels odd it can be written as a diagonal partition function in terms of the GSO-orbits under the β-vectors (2.7), which in this case have all equal length $2^r\,K$.

Let us also state the modular S-transformation rules for the characters involved in (2.9). For the $\mathrm{SU}(2)_k$ Kac-Moody algebra the S-matrix is given by

$$S_{l,l'} = \sqrt{\frac{2}{k+2}}\,\sin(l,l')_k\,, \tag{2.10}$$

where we have used the convention $(l,l')_k = \frac{\pi(l+1)(l'+1)}{k+2}$. For the N=2 minimal model, the modular S-matrix reads

$$\begin{aligned} S^{\mathrm{U}(1)_2}_{s_0,s_0'} &= \frac{1}{2}e^{-i\pi\frac{s_0 s_0'}{2}}\,, \\ \mathcal{S}_{(l,m,s),(l',m',s')} &= \frac{1}{2\sqrt{2k+4}}\,S_{l,l'}\,e^{i\pi\frac{m\,m'}{k+2}}\,e^{-i\pi\frac{s\,s'}{2}}\,. \end{aligned} \tag{2.11}$$

For a discussion of the normalization see [37].

The loop- and tree-channel Möbius amplitudes are related by the P-matrix $P = T^{\frac{1}{2}}S\,T^2\,S\,T^{\frac{1}{2}}$, which for just the $\mathrm{SU}(2)_k$ Kac-Moody algebra is given by

$$P_{l,l'} = \frac{2}{\sqrt{k+2}}\,\sin\frac{1}{2}(l,l')_k\,\delta^{(2)}_{l+l'+k,0} \tag{2.12}$$

and for the N=2 unitary models reads [29]

$$\begin{aligned} P^{\mathrm{U}(1)_2}_{s_0,s_0'} &= \frac{1}{\sqrt{2}}\,\sigma_{s_0}\sigma_{s_0'}\,e^{-i\pi\frac{s_0 s_0'}{4}}\,\delta^{(2)}_{s_0+s_0',0}, \\ \mathcal{P}_{(l,m,s),(l',m',s')} &= \frac{1}{2\sqrt{2k+4}}\,\sigma_{(l,m,s)}\,\sigma'_{(l',m',s')}\,e^{\frac{i\pi}{2}\frac{m\,m'}{k+2}}\,e^{-i\pi\frac{s\,s'}{4}}\,\delta^{(2)}_{s+s',0}\,\times \\ &\quad\times\left[P_{l,l'}\,\delta^{(2)}_{m+m'+k+2,0} + (-1)^{\frac{l'+m'+s'}{2}}\,e^{i\pi\frac{m+s}{2}}\,P_{l,k-l'}\,\delta^{(2)}_{m+m',0}\right]. \end{aligned} \tag{2.13}$$

The extra sign factors in (2.13),

$$\begin{aligned} \sigma_{s_0} &= (-1)^{h_{s_0}-\Delta_{s_0}} \\ \sigma_{(l,m,s)} &= (-1)^{h^l_{m,s}-\Delta^l_{m,s}} \end{aligned} \tag{2.14}$$

stem from the roots of the modular T-matrix in the definition of P.

Since in the following we restrict ourselves to the case of all levels being odd, we present in table 1 all Gepner models of this type and their corresponding Calabi-Yau manifold.

Apparently, for all levels odd the number of tensor factors is either five or nine. Therefore the formulas to be presented in the following sections are derived under the assumption of $r = 5, 9$ and all levels k_j odd. However, we would like to point out that we have evidence for some of them to hold also for the case of even levels [41].

levels	(h_{21}, h_{11})	CY
(1^9)	$(84, 0)$	–
$(1, 1, 3, 7, 43)$	$(67, 19)$	$\mathbb{P}_{1,5,9,15,15}[45]$
$(1, 1, 3, 13, 13)$	$(103, 7)$	$\mathbb{P}_{1,1,3,5,5}[15]$
$(1, 1, 5, 5, 19)$	$(65, 17)$	$\mathbb{P}_{1,3,3,7,7}[21]$
$(1, 1, 7, 7, 7)$	$(112, 4)$	$\mathbb{P}_{1,1,1,3,3}[9]$
$(1, 3, 3, 3, 13)$	$(75, 3)$	$\mathbb{P}_{1,3,3,3,5}[15]$
$(3, 3, 3, 3, 3)$	$(101, 1)$	$\mathbb{P}_{1,1,1,1,1}[5]$

Table 1: Odd level Gepner models.

3. Modular invariant partition functions from simple current construction

3.1 Review of general simple current extension

Recall that for a given conformal field theory there exists a very general way to construct modular invariant partition functions via an extension of the chiral symmetry algebra by some element of the set of simple currents [38]. These simple currents are primary fields J_i whose OPE with any other primary field ϕ_i only involves one particular other primary field, i.e.

$$J_a \times \phi_j = \phi_k \tag{3.1}$$

under fusion. It follows in particular, by associativity of the fusion rules, that the OPE of two simple currents yields again a simple current, so that in a rational CFT the set of simple currents forms a finite abelian group $\mathcal{S}$ under the fusion product. Denoting the length of the simple current J_a as $\mathcal{N}_a$, the set $\{J_a, J_a^2, \ldots, J_a^{\mathcal{N}_a}\}$ forms an abelian subgroup of $\mathcal{S}$ isomorphic to $\mathbb{Z}_{\mathcal{N}_a}$ with $(J_a^n)^C \equiv (J_a^n)^{-1} = J_a^{\mathcal{N}_a - n}$. Similarly, every simple current groups the primary fields into orbits $\{\phi_i, J_a \times \phi_i, J_a^2 \times \phi_i, \ldots, J_a^{\mathcal{N}_a^i - 1} \times \phi_i\}$ whose length $\mathcal{N}_a^i$ is a divisor of $\mathcal{N}_a$.

The crucial observation is that the action of simple currents in a RCFT implies the existence of a conserved quantity for every primary ϕ_i, the monodromy charge $Q_i^{(a)}$, defined by

$$J_a(z)\, \phi_i(w) = (z - w)^{-Q_i^{(a)}}\, \phi_k(w) + \cdots\,. \tag{3.2}$$

The monodromy of the identity being 1, it is clear that $Q_i^{(a)} = \frac{t_a^i}{\mathcal{N}_a} \bmod 1$ for some integer t_a^i. On the other hand, the monodromy is given by the conformal dimensions of the primary and the simple current as

$$Q_i^{(a)} = h(\phi_i) + h(J_a) - h(J_a \times \phi_i) \bmod 1\,, \tag{3.3}$$

from which one can show that

$$Q_i^{(a)}(J_a^n \times \phi_i) = \frac{t_a^i + r_a n}{\mathcal{N}_a} \bmod 1\,. \tag{3.4}$$

Here the monodromy parameter r_a is defined such that

$$h(J_a) = \frac{r_a(\mathcal{N}_a - 1)}{2\mathcal{N}_a} \bmod 1\,. \tag{3.5}$$

Inspired by the idea of orbifolding the CFT with respect to this world-sheet symmetry, one can prove that a simple current J_a with even monodromy parameter r_a induces the following even more general modular invariant partition function

$$Z_D(\tau,\bar\tau) = \sum_{k,l} \chi_k(\tau)\,(M_a)_{kl}\,\chi_l(\bar\tau)\,, \tag{3.6}$$

where

$$(M_a)_{kl} = \sum_{p=1}^{\mathcal{N}_a} \delta(\phi_k, J_a^p \times \phi_l)\,\delta^{(1)}(\hat{Q}^{(a)}(\phi_k) + \hat{Q}^{(a)}(\phi_l)) \tag{3.7}$$

and

$$\hat{Q}^{(a)}(\phi_i) = \frac{t_a^i}{2\mathcal{N}_a} \bmod 1\,. \tag{3.8}$$

Note that the proof relies on the fact that r_a is even, which can always be arranged for odd $\mathcal{N}_a$, with r_a being defined only mod$\mathcal{N}_a$. This yields two very different types of simple current invariants: If the conformal dimension of J_a is integer-valued, i.e. $r_a = 0 \bmod 1$, Z_D can be written as a left-right symmetric combination of orbits of primaries of integer monodromy, each occurring with multiplicity $\mathcal{N}_a/\mathcal{N}_a^i$. In all other cases, Z_D forms an automorphism type invariant in the sense that $Z_D = \sum_l \chi_l(\tau)\ \chi_{\Pi(l)}(\bar\tau)$ for some permutation Π of the primary labels. Furthermore, given two modular invariant matrices M_{a_1} and M_{a_2}, it is clear that $Z_D = \frac{1}{N}\sum_{k,l,m,} \chi_l\,(M_{a_1})_{lk}\,(M_{a_2})_{km}\,\chi_m$ is another modular invariant partition function with obvious generalizations for several M_{a_i}; the normalization factor N ensures that the vacuum appears precisely once in Z_D. As can be checked from above, $(M_a)^2 = 1$. The matrices M are also seen to commute if the respective simple currents J_a and J_b are mutually local, i.e. if their relative monodromy charge $Q^{(a)}(J_b) = 0 \bmod 1$, in which case the various δ-function constraints simplify considerably, as will turn out in our application to the Gepner model below.

The above procedure can equally well be used to construct simple current invariants for partition functions other than the diagonal invariant. In particular, one can verify that the extended charge conjugated partition function is obtained from

$$(M_a^C)_{kl} = \sum_{p=1}^{\mathcal{N}_a} \delta(\phi_k, (J_a^p \times \phi_l)^C)\,\delta^{(1)}(\hat{Q}^{(a)}(\phi_k) + \hat{Q}^{(a)}(\phi_l^C))\,. \tag{3.9}$$

3.2 Simple currents in the Gepner model

To begin with, let us identify the simple currents for the Gepner model. Given the fusion rules

$$\phi^0_{(m_1,s_1)} \times \phi^{l_2}_{(m_2,s_2)} = \phi^{l_2}_{(m_1+m_2,s_1+s_2)}\,, \tag{3.10}$$

we conclude that the simple currents J_α of the Gepner model under consideration can be labeled by the vector

$$j_\alpha = (s_0^\alpha; m_1^\alpha, \ldots, m_r^\alpha; s_1^\alpha, \ldots, s_r^\alpha)\,. \tag{3.11}$$

To compute the orbit length of the generic primary $\phi^{l_i}_{(m_i,s_i)}$ under J_α, we take into account that the m_i- and s_i- labels are only defined mod $(2k_i+4)$ and mod 4 respectively, so that for all levels odd (i.e. in the absence of short orbits resulting from the field identifications $(l_i,m_i,s_i)\cong(k_i-l_i,m_i+k_i+2,s_i+2)$) the orbit length $\mathcal{N}^\alpha_{\lambda,\mu}$ is the smallest positive integer such that

$$x_i^\alpha=\frac{\mathcal{N}^\alpha_{\lambda,\mu}}{2(k_i+2)}\,m_i^\alpha \qquad \text{and} \qquad y_j^\alpha=\frac{\mathcal{N}^\alpha_{\lambda,\mu}}{4}\,s_j^\alpha \tag{3.12}$$

are integer-valued numbers for all i,j from $\{1,\ldots,r\}$ and $\{0,\ldots,r\}$ respectively. It is clear that the orbit length only depends on the simple current in question and is the same for every primary, $\mathcal{N}^\alpha_{\lambda,\mu}\equiv\mathcal{N}^\alpha$.

As indicated above, the simultaneous extension of the chiral algebra by several simple currents is possible as long as they are mutually local. For the construction to make sense, we therefore need to impose first compatibility with the simple currents (2.7) generating the GSO-like Gepner projection, i.e.

$$\begin{array}{lll} J_0 & \text{with} & j_0\equiv\beta_0=(1;1,\ldots,1;1,\ldots,1) \\ J_i & \text{with} & j_i\equiv\beta_i=(2;0,\ldots,0;0,\ldots,\underbrace{2}_{i-th},\ldots,0)\,. \end{array} \tag{3.13}$$

Starting with the latter and using (3.3) together with (2.3) for the computation of the monodromy charge, this yields the constraints

$$Q^{(\alpha)}(J_i)=-\frac{s_0^\alpha}{2}-\frac{s_i^\alpha}{2}=0 \bmod 1\,, \tag{3.14}$$

so that $s_0^\alpha+s_i^\alpha=0 \mod 2$ for $i=1,\ldots r$. Without loss of generality, we can restrict ourselves to the case of even s_0^α, s_i^α and therefore also even m_i^α, as required by the usual constraint that $l_i^\alpha+m_i^\alpha+s_i^\alpha=0 \mod 2$. The point is that this is not really a restriction of the generality of the currents to be used since currents from the Neveu-Schwarz sector are related to those in the Ramond sector by spectral flow, of course.

Furthermore, we can even assume for the following reason that actually $s_i^\alpha=0 \mod 4$. Since we will eventually be summing over the orbit generated by J_i and J_α in the partition function independently (cf. (3.20) and (3.22) below), a non-zero value of s_i^α can be compensated for by the corresponding contribution from J_i. In other words, we can encode the information about the value of s_j^α by a shift of s_0^α, because with the above procedure the two currents $(s_0^\alpha;m_j^\alpha;2)$ and $(s_0^\alpha+2;m_j^\alpha;0)$ are equivalent.

Mutual locality with the Gepner current J_0 necessitates that

$$Q^{(\alpha)}(J_0)=\sum_{i=1}^r\left(\frac{m_i^\alpha}{2(k_i+2)}\right)-\frac{s_0^\alpha}{4}=0 \bmod 1\,. \tag{3.15}$$

For all levels k_i odd, this can only be satisfied if $s_0^\alpha=0 \mod 4$. To summarize, the most general set of simple currents consistent with the Gepner extension is labeled by the vector

$$j_\alpha=(0;m_1^\alpha,\ldots,m_r^\alpha;0,\ldots,0) \tag{3.16}$$

for even m_i^α. In view of (3.12), this means that the orbit length is odd for every simple current under consideration.

Furthermore, compatibility of the simple currents with each other imposes the constraint

$$Q_{\alpha,\beta} \equiv Q^{(\alpha)}(J_\beta) = \sum_{i=1}^{r} \left(\frac{m_i^\alpha \, m_i^\beta}{2(k_i+2)} \right) = 0 \bmod 1 \qquad \text{for } \alpha \neq \beta\,. \tag{3.17}$$

Finally, it is sufficient to consider the cases in which none of the simple currents lies in the cyclic group generated by any of the other simple currents or of an arbitrary product of them. As pointed out above, we do not miss any generically new cases by this requirement, since, e.g., $M(J_\alpha)\, M(J_\alpha^n) = 1$ for all $n \neq 0$.

3.3 Diagonal and charge conjugate partition functions for Gepner models

We are now in a position to construct the diagonal invariant for the Gepner model extended by appropriate currents $J_\alpha, \alpha = 1, \ldots, I$. Since $\mathcal{N}_\alpha$ is odd, J_α and J_α^2 generate the same orbit when acting on the primary ϕ^λ_μ, which may therefore be parametrized as $\{\phi^\lambda_{\mu+2\tau_\alpha j_\alpha}\}$ with $\tau_\alpha = 0, \ldots, \mathcal{N}_\alpha - 1$.

To deal with the δ-functions appearing in the extended partition function, we observe that the arguments of the $\delta^{(1)}$-functions as written in (3.7) are of the form

$$\begin{aligned} \hat{Q}^{(\alpha)} \left(\phi^\lambda_{\tilde{\mu}+2\,\tau_\alpha j_\alpha} \right) + \hat{Q}^{(\alpha)} \left(\phi^\lambda_{\tilde{\mu}} \right) &= Q^{(\alpha)} \left(\phi^\lambda_{\tilde{\mu}} \right) + 2\,\tau_\alpha \hat{Q}^{(\alpha)}(J_\alpha) \\ &= \left(Q^{(\alpha)} \left(\phi^\lambda_{\tilde{\mu}} \right) + 2\,\tau_\alpha \frac{r_\alpha}{2\mathcal{N}_\alpha} \right) \bmod 1\,, \end{aligned} \tag{3.18}$$

where $\tilde{\mu} = \mu + \sum_{\beta\neq\alpha} 2\,\tau_\beta\, j_\beta$ accounts for the twists from the simple currents other than τ_α. A straightforward calculation shows that due to mutual locality of the currents

$$Q^{(\alpha)} \left(\phi^\lambda_{\mu+\sum_{\beta\neq\alpha} 2\,\tau_\beta\, j_\beta} \right) = Q^{(\alpha)} \left(\phi^\lambda_\mu \right) \bmod 1\,. \tag{3.19}$$

Putting things together results in the following extension of the diagonal invariant partition function (2.9)

$$\begin{aligned} Z_D(\tau,\bar{\tau}) \;=\; & \frac{1}{N}\frac{1}{2^r}\frac{(\mathrm{Im}\tau)^{-2}}{|\eta(q)|^4} \sum_{b_0=0}^{K-1} \sum_{b_1,\ldots,b_r=0}^{1} \sum_{\tau_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\tau_I=0}^{\mathcal{N}_I-1} \sum_{\lambda,\mu}^{\beta} (-1)^{s_0} \times \\ & \times \prod_{\alpha=1}^{I} \delta^{(1)} \left(Q^{(\alpha)}_{\lambda,\mu} + 2\,\tau_\alpha \hat{Q}^\alpha(J_\alpha) \right) \chi^\lambda_\mu(q)\; \chi^\lambda_{\mu+b_0\beta_0+b_1\beta_1+\ldots+b_r\beta_r+\sum_\alpha 2\,\tau_\alpha j_\alpha}(\bar{q})\,, \end{aligned} \tag{3.20}$$

where

$$Q^{(\alpha)}_{\lambda,\mu} = Q^{(\alpha)}(\phi^\lambda_\mu) = \sum_i \left(\frac{m_i\, m_i^\alpha}{2(k_i+2)} \right) \bmod 1\,. \tag{3.21}$$

Likewise, one obtains a similar expression for the charge conjugate partition function

$$\begin{aligned} Z_C(\tau,\bar{\tau}) \;=\; & \frac{1}{N}\frac{1}{2^r}\frac{(\mathrm{Im}\tau)^{-2}}{|\eta(q)|^2} \sum_{b_0=0}^{K-1} \sum_{b_1,\ldots,b_r=0}^{1} \sum_{\tau_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\tau_I=0}^{\mathcal{N}_I-1} \sum_{\lambda,\mu}^{\beta} (-1)^{s_0} \times \\ & \times \prod_{\alpha=1}^{I} \delta^{(1)} \left(Q^{(\alpha)}_{\lambda,-\mu} + 2\,\tau_\alpha \hat{Q}^{(\alpha)}(J_\alpha) \right) \chi^\lambda_\mu(q)\; \chi^\lambda_{-\mu+b_0\beta_0+b_1\beta_1+\ldots+b_r\beta_r+\sum_\alpha 2\,\tau_\alpha j_\alpha}(\bar{q})\,. \end{aligned} \tag{3.22}$$

4. Orientifolds of extended Gepner models: the A-type Klein bottle

A-type orientifolds are obtained by projecting the bulk theory as defined by the charge conjugate partition function onto Ω-invariant states, i.e. onto those states coupling symmetrically in Z_C. From (3.22) it is evident that this projection requires for states appearing in the Klein bottle K^A

$$\mu \cong -\mu + b_0\beta_0 + b_1\beta_1 + \ldots + b_r\beta_r + \sum_\alpha 2\,\tau_\alpha j_\alpha\,, \tag{4.1}$$

i.e.

$$m_j = b + \sum_\alpha \tau_\alpha m_j^\alpha \text{ mod } (k_j+2) \qquad \text{for all } j \tag{4.2}$$

for some b in the range $\{0,\ldots,\frac{K}{2}-1\}$ and where τ_α is as usual from $\{0,\ldots,\mathcal{N}_\alpha-1\}$ but has to satisfy in addition the δ-constraints from Z_C, i.e. is actually a function $\tau_\alpha(\lambda,\mu)$.

To incorporate these projections correctly, it is convenient to read off the δ-constraints directly from the very general from of the simple current extended conjugate partition function (3.9) as

$$\prod_{\gamma=1}^{I} \delta^{(1)}\left(\hat{Q}^{(\gamma)}_{-\mu+\sum_{i=0}^r b_i\beta_i + \sum_{\alpha=1}^{\gamma-1} 2\tau_\alpha j_\alpha} + \hat{Q}^{(\gamma)}_{-\mu+\sum_{i=0}^r b_i\beta_i + \sum_{\alpha=1}^{\gamma} 2\tau_\alpha j_\alpha}\right). \tag{4.3}$$

Under the Klein bottle projection (4.1), a generic δ-constraint is seen to equal

$$\begin{aligned}
&\delta^{(1)}\left(\hat{Q}^{(\gamma)}_{-\mu+\sum_{i=0}^r b_i\beta_i + \sum_{\alpha=1}^{\gamma-1} 2\tau_\alpha j_\alpha} + \hat{Q}^{(\gamma)}_{-\mu+\sum_{i=0}^r b_i\beta_i + \sum_{\alpha=1}^{\gamma} 2\tau_\alpha j_\alpha}\right) = \\
&\quad = \delta^{(1)}\left(\hat{Q}^{(\gamma)}_{-\mu+\sum_{i=0}^r b_i\beta_i + \sum_{\alpha=1}^{\gamma-1} 2\tau_\alpha j_\alpha} + \hat{Q}^{(\gamma)}_{\mu-\sum_{\alpha=\gamma+1}^{I} 2\tau_\alpha j_\alpha}\right) \\
&\quad = \delta^{(1)}\left(\sum_{i=0}^r b_i \hat{Q}^{(\gamma)}(J_i) + \sum_{\alpha=1}^{\gamma-1}\tau_\alpha Q^{(\gamma)}(J_\alpha) - \sum_{\alpha=\gamma+1}^{I}\tau_\alpha Q^{(\gamma)}(J_\alpha)\right).
\end{aligned} \tag{4.4}$$

Two things are crucial to observe: First, the projection on states with integer monodromy drops out completely, since, of course, $\hat{Q}^{(\gamma)}_\mu + \hat{Q}^{(\gamma)}_{-\mu} = 0$. Second, and most importantly, the (possibly non-integer) monodromy charges of the currents with themselves, $Q^{(\alpha)}(J_\alpha)$, do not occur in any of the arguments either. As for the hatted monodromies of the Gepner currents, it is clear that the monodromy charge of the currents J_i, $i=1,\ldots,r$ vanishes anyway because s_i^α vanishes mod 4 for all i. The Klein bottle projection (4.1) implies furthermore that only even values of b_0 contribute, so that for J_0 we are left with the unhatted monodromy charge in (4.4) as well, which is integer a priori.

Therefore, the monodromy projections are satisfied identically and we are left with the following A-type Klein bottle

$$K^A = 4\int_0^\infty \frac{dt}{t^3}\frac{1}{2^{r+1}}\frac{1}{\eta(2it)^2}\sum_{\lambda,\mu}^{\beta}\sum_{b=0}^{\frac{K}{2}-1}\sum_{\tau_1=0}^{\mathcal{N}_1-1}\ldots\sum_{\tau_I=0}^{\mathcal{N}_I-1}\prod_{j=1}^{r}\delta^{(k_j+2)}_{m_j,b+\sum_\alpha \tau_\alpha m_j^\alpha}(-1)^{s_0}\,\chi^\lambda_\mu(2it)\,. \tag{4.5}$$

Transforming this into tree channel with the methods of [37] yields

$$\tilde{K}^A = \frac{2^4(\prod_{\alpha=1}^{I}\mathcal{N}_\alpha)}{2^{\frac{3r}{2}}\prod_j\sqrt{k_j+2}}\int_0^\infty dl\frac{1}{\eta^2(2il)}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{K-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1}(-1)^{\nu_0}\times$$
$$\times\delta^{(4)}_{s_0'+\nu_0+2\sum\nu_j+2,0}\,\delta^{(K')}_{\sum_j\frac{K'}{2k_j+4}(m_j'+\nu_0+(1-\epsilon_j)(k_j+2)),0}\left(\prod_\alpha\delta^{(1)}(Q^{(\alpha)}_{\lambda',\mu'})\right)\times$$
$$\times\prod_{j=1}^{r}\left(\frac{P^2_{l_j',\epsilon_j k_j}}{S_{l_j',0}}\,\delta^{(2)}_{m_j'+\nu_0+(1-\epsilon_j)(k_j+2),0}\,\delta^{(4)}_{s_j'+\nu_0+2\nu_j+2(1-\epsilon_j),0}\right)\chi^{\lambda'}_{\mu'}(2il)\,, \tag{4.6}$$

with $K'=\mathrm{lcm}(k_j+2)$. The crosscap state can be extracted from (4.6) up to additional sign factors

$$|C\rangle_A = \frac{1}{\kappa_c^A}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{K-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1}\Sigma(\lambda',\mu',\nu_0,\nu_j,\epsilon_j,m_j^\alpha)\times$$
$$\times\delta^{(4)}_{s_0'+\nu_0+2\sum\nu_j+2,0}\,\delta^{(K')}_{\sum_j\frac{K'}{2k_j+4}(m_j'+\nu_0+(1-\epsilon_j)(k_j+2)),0}\left(\prod_\alpha\delta^{(1)}(Q^{(\alpha)}_{\lambda',\mu'})\right)\times$$
$$\times\prod_{j=1}^{r}\left(\frac{P_{l_j',\epsilon_j k_j}}{\sqrt{S_{l_j',0}}}\,\delta^{(2)}_{m_j'+\nu_0+(1-\epsilon_j)(k_j+2),0}\,\delta^{(4)}_{s_j'+\nu_0+2\nu_j+2(1-\epsilon_j),0}\right)|\lambda',\mu'\rangle\rangle_c\,, \tag{4.7}$$

with

$$\left(\frac{1}{\kappa_c^A}\right)^2 = \frac{2^5(\prod_{\alpha=1}^{I}\mathcal{N}_\alpha)}{2^{\frac{3r}{2}}K\prod_j\sqrt{k_j+2}}\,. \tag{4.8}$$

For completeness, we give a similar derivation of the B-type Klein bottle in the appendix.

5. The A-type Annulus amplitude

As we can see from the various constraints in (4.6), it is the states coupling diagonally in Z_D which contribute to the divergent A-type Klein bottle amplitude. For one-loop consistency of the string spectrum we therefore need to introduce an appropriate amount of D-branes, i.e. A-type RS boundary states canceling the divergent terms from the orientifold plane. The A-type boundary states of the pure Gepner model read

$$|a\rangle_A = |S_0;(L_j,M_j,S_j)_{j=1}^r\rangle_A = \frac{1}{\tilde{\kappa}_a^A}\sum_{\lambda',\mu'}^{\beta}(-1)^{\frac{s_0'^2}{2}}e^{-i\pi\frac{s_0'S_0}{2}}\times$$
$$\times\prod_{j=1}^{r}\left(\frac{S_{l_j',L_j}}{\sqrt{S_{l_j',0}}}e^{i\pi\frac{m_j'M_j}{k_j+2}}\,e^{-i\pi\frac{s_j'S_j}{2}}\right)|\lambda',\mu'\rangle\rangle\,, \tag{5.1}$$

where we use the normalization as computed in [37]

$$\frac{1}{(\tilde{\kappa}_a^A)^2} = \frac{K}{2^{\frac{r}{2}+1}\prod_j\sqrt{k_j+2}}\,. \tag{5.2}$$

The authors of [18] pointed out that two boundary states of a theory whose symmetry algebra is extended by a certain group of simple currents are equivalent if they lie in the same orbit under a simple current. Applied to the pure Gepner case, this means that the boundary states related by the action of the simple currents J_0 and J_i are equivalent, which is consistent also with the detailed form of the various open string amplitudes as calculated in [37]. Together with the constraint $L_j + M_j + S_j = 0 \bmod 2$ and the reflection symmetry $(L_j, M_j, S_j) \to (k_j - L_j, M_j + k_j + 2, S_j + 2)$ this allows us to bring the independent boundary states to the form $|S_0; (L_j, M_j, 0)_{j=1}^r\rangle_A$ with $L_j = M_j = 0 \bmod 2$. Recall from [37] that boundary states with odd S_j are excluded in the orientifold, being inconsistent with the Möbius amplitude, since the crosscap state is formally given by $|0, (L_j, 0, 0)_{j=1}^r\rangle$ for certain numbers L_j and all tensor factors need to lie either in the NS- or the R-sector.

In this paper we impose the stronger condition that all boundary states should also be relatively supersymmetric with respect to the orientifold plane, thus giving rise to the additional condition

$$\sum_{j=1}^{r} \frac{M_j}{k_j+2} - \sum_{j=1}^{r} \frac{S_j}{2} - \frac{S_0}{2} = 0 \bmod 2\,. \tag{5.3}$$

With all levels being odd, this implies $S_0 = 0$ with $S_0 = 2$ describing the anti-branes. Therefore, the independent supersymmetric A-type boundary states of the pure Gepner model are given by the boundary states

$$|0; (L_j, M_j, 0)_{j=1}^r\rangle_A \qquad \text{with } L_j, M_j \text{ even}\,. \tag{5.4}$$

For a Gepner model extended by additional simple currents the boundary states of the new model are given by orbits under the simple current actions of the boundary states of the pure Gepner model [15]

$$|a, J_1, \ldots, J_I\rangle_A = \frac{1}{\sqrt{\prod_\alpha \mathcal{N}_\alpha}} \sum_{\tau_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\tau_I=0}^{\mathcal{N}_I-1} \prod_{\alpha=1}^{I} J_\alpha^{\tau_\alpha} |a\rangle_A\,. \tag{5.5}$$

Therefore, now the independent boundary states are labeled by simple current orbits of the pure Gepner model boundary states. For simplicity, in the following we will label them by one of the representatives appearing in the respective simple current orbit. Inserting the Gepner model boundary states (5.1) into (5.5), the latter ones can also be written as

$$\begin{aligned} |a\rangle_A = |S_0; (L_j, M_j, S_j)_{j=1}^r\rangle_A = {} & \frac{1}{\kappa_a^A} \sum_{\lambda',\mu'}^{\beta} \prod_\alpha \delta^{(1)}(Q^{(\alpha)}_{\lambda',\mu'}) (-1)^{\frac{s_0'^2}{2}} e^{-i\pi \frac{s_0' S_0}{2}} \times \\ & \times \prod_{j=1}^{r} \left(\frac{S_{l_j', L_j}}{\sqrt{S_{l_j',0}}} e^{i\pi \frac{m_j' M_j}{k_j+2}} e^{-i\pi \frac{s_j' S_j}{2}} \right) |\lambda', \mu'\rangle\rangle \end{aligned} \tag{5.6}$$

with the normalization

$$\frac{1}{(\kappa_a^A)^2} = \frac{K\ (\prod_\alpha \mathcal{N}_\alpha)}{2^{\frac{r}{2}+1} \prod_j \sqrt{k_j+2}}\,. \tag{5.7}$$

Therefore, in the boundary states only A-type-Ishibashi states with integer monodromy charge appear. This is consistent with the analogous statement for the crosscap states, which clearly holds in view of (4.7).

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Forming the overlap between two stacks of branes $|a\rangle$ and $|\tilde{a}\rangle$ with Chan-Paton multiplicities N_a and $N_{\tilde{a}}$ respectively results in the tree-channel annulus amplitude

$$\tilde{A}^A_{\tilde{a}a} = \frac{N_a N_{\tilde{a}}}{\kappa^A_a \, \kappa^A_{\tilde{a}}} \int_0^\infty \frac{dl}{\eta^2(2il)} \sum^{\beta}_{\lambda',\mu'} \prod_\alpha \delta^{(1)}(Q^{(\alpha)}_{\lambda',\mu'})(-1)^{s_0'^2} e^{-i\pi\frac{s_0'}{2}(S_0-\tilde{S}_0)} \times$$
$$\times \prod_{j=1}^{r} \left(\frac{S_{l_j',L_j} \, S_{l_j',\tilde{L}_j}}{S_{l_j',0}} \, e^{i\pi\frac{m_j'(M_j-\tilde{M}_j)}{k_j+2}} \, e^{-i\pi\frac{s_j'(S_j-\tilde{S_j})}{2}} \right) \chi^{\lambda'}_{\mu'}(2il) \,. \tag{5.8}$$

In order to finally read off the massless spectrum, we have to transform into loop channel

$$A^A_{\tilde{a}\,a} = N_a N_{\tilde{a}} \, \frac{1}{2^{r+1}} \int_0^\infty \frac{dt}{t^3} \frac{1}{\eta^2(it)} \sum^{ev}_{\lambda,\mu} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \sum_{\sigma_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\sigma_I=0}^{\mathcal{N}_I-1} (-1)^{\nu_0} \times$$
$$\times \delta^{(4)}_{s_0,2+\tilde{S}_0-S_0-\nu_0-2\sum_j \nu_j} \prod_{j=1}^{r} \times \tag{5.9}$$
$$\times \left(N^{|\epsilon_j k_j - l_j|}_{L_j,\tilde{L}_j} \, \delta^{(2k_j+4)}_{m_j+M_j-\tilde{M}_j+\nu_0+\sum_\alpha \sigma_\alpha m_j^\alpha+\epsilon_j(k_j+2),0} \, \delta^{(4)}_{s_j,\tilde{S}_j-S_j-\nu_0-2\nu_j+2\epsilon_j} \right) \chi^\lambda_\mu(it) \,.$$

6. The A-type Möbius amplitude

In [37], a strategy to determine the signs of the crosscap states was developed: Examining the transformation of a whole orbit generated by simple currents (in the case studied there, the Gepner currents) under world-sheet duality, it turned out to be possible to fix the signs of $|C\rangle$ by requiring that each orbit transform exactly into another full orbit. This yields the Möbius amplitude as the overlap between crosscap and RS boundary states up to an overall sign, which, as is usually done in constructing the open string sector, is fixed a posteriori by the tadpole conditions. Following this general philosophy, it is important to take the distinction between simple currents of integer and non-integer conformal dimension into consideration. Recall that the latter act as automorphisms on the set of primaries in the diagonal invariant, as described in section 3, so that the actual orbits are generated only by the integer spin currents and, of course, the currents inducing the Gepner projection as before. Suppose therefore that $h(J_\alpha) = 0 \mod 1$ for all α in $1, \ldots, I'$. We are then interested in the P-transformation of the hypothetical Neveu-Schwarz sector Möbius amplitude

$$M^\lambda_\mu = \sum_{\nu_0=0}^{\frac{K}{2}} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\tau_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\tau_{I'}=0}^{\mathcal{N}_{I'}-1} (-1)^{[h^\lambda_\mu(\nu_0,\nu_j,\tau_\alpha)-h^\lambda_\mu]} \hat{\chi}^\lambda_{\mu+2\nu_0\beta_0+\sum \nu_j\beta_j+\sum_{\alpha=1}^{I'} 2\tau_\alpha m_j^\alpha} \left(it+\frac{1}{2}\right), \tag{6.1}$$

where $h^\lambda_\mu(\nu_0,\nu_j,\tau_\alpha)$ are the conformal dimensions of the states appearing in the orbit and $h^\lambda_\mu = h^\lambda_\mu(0,0,0)$ and the extra signs go back to writing the amplitude in terms of the hatted characters. The latter ones are defined as

$$\hat{\chi}\left(it+\frac{1}{2}\right) = e^{-i\pi\left(h-\frac{c}{24}\right)} \chi\left(it+\frac{1}{2}\right) . \tag{6.2}$$

Note that the primary ϕ^λ_μ from which the orbit is generated has to appear in the partition function, of course, to be eligible for the Möbius amplitude at all, i.e. it has to satisfy the Gepner constraints and must have integer monodromy with the integer-spin simple currents. The only open question is in as far the simple current contribution to the total conformal dimension of the states modifies the resummation of the P-transformation. As is shown in the appendix, the signs turn out to be unchanged as compared to [37]. In particular, the orbit-into-orbit condition is seen to be satisfied correctly, including the requirement of integer monodromy of the orbits, and we find eventually

$$
\begin{aligned}
\tilde{M}^\lambda_\mu \sim \sum_{\lambda',\mu'}^{\beta} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} & \left(\prod_{\alpha=1}^{I'} \delta^{(1)}(Q^{(\alpha)}_{\lambda',\mu'})\right)\left(\prod_{j=1}^{r} \sigma_{(l'_j,m'_j,s'_j)}\right)\left(\prod_{k<l}(-1)^{\eta_k\eta_l}\right)\delta^{(2)}_{\sum_j \eta_j,0}\, e^{-i\pi\frac{s_0 s'_0}{4}} \times \\
& \times \delta^{(2)}_{s_0+s'_0,0} \prod_{j=i}^{r}\Bigg(P_{l_j,|\epsilon_j k_j - l'_j|}\, e^{i\pi\frac{m_j m'_j}{2k_j+4}}\, \delta^{(2)}_{m_j+m'_j+(1-\epsilon_j)(k_j+2),0}\, e^{-i\pi\frac{s_j s'_j}{4}}\, \delta^{(2)}_{s_j+s'_j,0} \times \\
& \qquad \times (-1)^{\epsilon_j\frac{m_j+s_j}{2}}\,(-1)^{\epsilon_j\frac{l'_j+m'_j+s'_j}{2}}\Bigg)\hat{\chi}^{\lambda'}_{\mu'}\left(2il+\frac{1}{2}\right),
\end{aligned}
\tag{6.3}
$$

where

$$
\eta_j = \frac{s_0+s_j}{2} - \frac{s'_0+s'_j}{2} + \epsilon_j + 1\,. \tag{6.4}
$$

Extracting the signs from (6.3) and forming the overlap between a crosscap and RS boundary state in the NS-sector yields for the A-type

$$
\begin{aligned}
\tilde{M}^{A,NS}_a = & -\frac{2N_a}{\kappa^A_C\kappa^A_a}\int_0^\infty dl\,\frac{1}{\hat{\eta}^2(2il+\frac{1}{2})}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{\frac{K}{2}-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \times \\
& \times \prod_\alpha \delta^{(1)}\left(Q^{(\alpha)}_{\lambda',\mu'}\right)(-1)^{\nu_0}\left(\prod_{k<l}(-1)^{\nu_k\nu_l}\right)(-1)^{\sum_j \nu_j}\, e^{-i\pi\frac{s'_0 S_0}{2}}\,\delta^{(4)}_{s'_0+2\nu_0+2\sum\nu_j+2,0} \times \\
& \times \delta^{(K')}_{\sum_j \frac{K'}{2k_j+4}(m'_j+2\nu_0+(1-\epsilon_j)(k_j+2)),0} \times \\
& \times \prod_{j=1}^{r}\Bigg(\sigma_{(l'_j,m'_j,s'_j)}\frac{P_{l'_j,\epsilon_j k_j} S_{l'_j,L_j}}{S_{l'_j,0}}\,\delta^{(2)}_{m'_j,\,2\nu_0+(1-\epsilon_j)(k_j+2)} \times \\
& \qquad \times \delta^{(4)}_{s'_j,\,2\nu_0+2\nu_j+2(1-\epsilon_j)}(-1)^{\epsilon_j\frac{m'_j+s'_j}{2}}\, e^{-i\pi\frac{m'_j M_j}{k_j+2}}\, e^{i\pi\frac{s'_j S_j}{2}}\Bigg)\chi^{\lambda'}_{\mu'}\left(2il+\frac{1}{2}\right).
\end{aligned}
\tag{6.5}
$$

A lengthy calculation gives the following loop-channel Möbius amplitude

$$
\begin{aligned}
M^{A,NS}_a = & (-1)^s N_a\,\frac{1}{2^{r+1}}\int_0^\infty \frac{dt}{t^3}\frac{1}{\hat{\eta}^2(it+\frac{1}{2})}\sum_{\lambda,\mu}^{ev}\sum_{\nu_0=0}^{\frac{K}{2}-1}\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1}\sum_{\sigma_1=0}^{\mathcal{N}_1-1}\cdots\sum_{\sigma_I=0}^{\mathcal{N}_I-1} \times \\
& \times\left(\prod_{k<l}(-1)^{\rho_k\rho_l}\right)\delta^{(2)}_{\sum_j \rho_j,0}\,\delta^{(2)}_{s_0,0} \times \\
& \times \prod_{j=1}^{r}\Bigg(\sigma_{(l_j,m_j,s_j)} Y^{l_j}_{L_j,\epsilon_j k_j}\,\delta^{(2k_j+4)}_{2M_j+m_j+2\nu_0+\sum_\alpha \sigma_\alpha m^\alpha_j-\epsilon_j(k_j+2),0} \times
\end{aligned}
\tag{6.6}
$$

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$$\times\,\delta^{(2)}_{s_j,0}\,(-1)^{\frac{\epsilon_j}{2}[2S_j-s_j-2\epsilon_j]}\,(-1)^{\frac{(1-\epsilon_j)}{2}[2M_j-m_j-\epsilon_j(k_j+2)]}\Big)\;\hat{\chi}^{\lambda}_{\mu}\left(it+\frac{1}{2}\right),$$

where

$$r\;=\;4s+1\,, \tag{6.7}$$

$$\rho_j\;=\;\frac{s_0+s_j}{2}+\epsilon_j-1 \tag{6.8}$$

and the Y-tensor is defined as

$$Y^{l_3}_{l_1,l_2}=\sum_{l=0}^{k}\frac{S_{l_1,l}\,P_{l_2,l}\,P_{l_3,l}}{S_{0,l}}\,. \tag{6.9}$$

7. Tadpoles and massless spectra

7.1 Tadpole cancellation conditions

The massless states lead to divergent terms in the Klein bottle, the annulus and the Möbius amplitude. In light-cone gauge, these are known to be those states with conformal dimension $h=1/2$. Recall that in the $N=2$ Super-Virasoro minimal model h is bounded from below by half the U(1)-charge, with equality holding exactly for chiral primaries, so we conclude that the A- type divergent terms stem at most from the fields in the (c,c)-ring. These are exactly the states

$$(2)\;(0,0,0)^5\quad\text{as well as}\quad(0)\;\prod_j(l_j;l_j;0)\qquad\text{with}\;\sum_j\frac{l_j}{k_j+2}=1\,. \tag{7.1}$$

Besides, the concrete formulas for the various amplitudes put further constraints on the chiral fields to actually contribute.

As in the pure Gepner case, we introduce stacks of N_a A-type RS -boundary states $|0;\prod_j(L^a_j,M^a_j,0)\rangle$ and also their Ω-image $|0;\prod_j(L^a_j,-M^a_j,0)\rangle$. One can then check the δ-function constraints in each of the tree-channel A-type amplitudes separately, assuming w.l.o.g that m'_j is even for the Ishibashi states by reflection symmetry $(l'_j,l'_j,0)\cong(k_j-l'_j,l'_j+k_j+2,2)$. The result is that only the above chiral fields satisfying in addition

$$Q^{(\alpha)}_{\lambda',\mu'}=0\bmod 1 \tag{7.2}$$

give a non-vanishing contribution. This is the net effect of the simple current extension and reduces the number of tadpole conditions to be satisfied. The actual tadpole conditions as such are unaltered as compared to the pure Gepner case [37] and take the amazingly simple form

$$\left(\sum_{a=1}^{N}2\,N_a\,\cos\left[\pi\sum_j\tfrac{m_j\,M^a_j}{k_j+2}\right]\prod_j\sin(l_j,L^a_j)_{k_j}-4\prod_j\sin\frac{1}{2}(l_j,k_j)_{k_j}\right)^2=0\,. \tag{7.3}$$

From the general tadpole cancellation conditions (7.3) it is immediately clear that there always exists a simple solution to these equations, namely by choosing one stack of D-branes with

$$L_j = \frac{k_j \mp 1}{2}, \qquad M_j = 0 \tag{7.4}$$

for all j and $k_j = 4n_j \pm 1$. The Chan-Paton factor is just $N_1 = 4$ and for $r = 5$ leads to a gauge group SO(4) and for $r = 9$ to SP(4). The interpretation of this solution is that we have just placed appropriate D-branes right on top of the orientifold plane.

7.2 The gauge sector

From the loop channel annulus and Möbius strip amplitudes it is a straightforward exercise to compute the massless spectrum. Recall that for each boundary state $|a\rangle = |0; \prod_j (L_j^a, M_j^a, 0)\rangle$ we have to introduce its Ω image $|a'\rangle = |-0; \prod_j (L_j^a, -M_j^a, 0)\rangle$. For each pair of boundary states we have to determine the number of massless states in the corresponding loop channel amplitudes.

Gauge fields only arise from open strings stretched between the same D-branes, as only then does the vacuum state

$$(2)\,(0,0,0)^5 \tag{7.5}$$

appear in A_{aa}. If the boundary state $|a\rangle$ is not invariant under Ω but mapped to a different state $|a'\rangle$, this pair of branes carries a $\mathrm{U}(N_a)$ gauge field on its world-volume. Consistently, in this case the Möbius strip amplitude M_a does not contain the vacuum state.

If however the boundary state is invariant under Ω, the vacuum state does arise in the Möbius strip amplitude M_a and depending on the sign one obtains a gauge field of either $\mathrm{SO}(2N_a)$ or $\mathrm{SP}(2N_a)$.

7.3 The matter sector

Additional massless matter can arise from all possible intersections of the boundary states. One has to compute how many massless states of the form

$$(0) \prod_j (l_j; l_j; 0) \qquad \text{respectively} \qquad (0) \prod_j (l_j; -l_j; 0) \tag{7.6}$$

with $(h, q) = (1/2, \pm 1)$ contribute to the annulus and Möbius strip amplitudes. More concretely, the various open string sectors give rise to the chiral massless matter spectrum shown in table 2.

Here we have defined the net number of generations by certain "topological" indices which correspond to the topological intersection number in the intersecting brane world models and from the world-sheet point of view to the Witten index in the corresponding open string sector respectively. Note that for the (anti-)symmetric representations the net number of generations is given by the following combination of indices

$$\begin{aligned} n^+_{a,\mathrm{S}} - n^-_{a,\mathrm{S}} &= \frac{1}{2}(I_{a'a} - I_{oa}) \\ n^+_{a,\mathrm{A}} - n^-_{a,\mathrm{A}} &= \frac{1}{2}(I_{a'a} + I_{oa})\,. \end{aligned} \tag{7.7}$$

sector	(h,q)	rep.	index
A_{ab}	$(\frac{1}{2},1)$	$n^+_{ab}\times(\overline{\mathbf{N}}_a,\mathbf{N}_b)$	
	$(\frac{1}{2},-1)$	$n^-_{ab}\times(\mathbf{N}_a,\overline{\mathbf{N}}_b)$	$I_{ab}=n^+_{ab}-n^-_{ab}$
$A_{a'b}$	$(\frac{1}{2},1)$	$n^+_{a'b}\times(\mathbf{N}_a,\mathbf{N}_b)$	
	$(\frac{1}{2},-1)$	$n^-_{a'b}\times(\overline{\mathbf{N}}_a,\overline{\mathbf{N}}_b)$	$I_{a'b}=n^+_{a'b}-n^-_{a'b}$
$A_{a'a}+M_a$	$(\frac{1}{2},1)$	$n^+_{a,\mathrm{S}}\times\mathbf{S}_a+n^+_{a,\mathrm{A}}\times\mathbf{A}_a$	$I_{a'a}=(n^+_{a,\mathrm{S}}+n^+_{a,\mathrm{A}})-(n^-_{a,\mathrm{S}}+n^-_{a,\mathrm{A}})$
	$(\frac{1}{2},-1)$	$n^-_{a,\mathrm{S}}\times\overline{\mathbf{S}}_a+n^-_{a,\mathrm{A}}\times\overline{\mathbf{A}}_a$	$I_{oa}=(n^+_{a,\mathrm{S}}-n^+_{a,\mathrm{A}})-(n^-_{a,\mathrm{S}}-n^-_{a,\mathrm{A}})$

Table 2: Massless chiral matter spectrum.

The index I_{oa} can be considered as the intersection number between the D-brane $|a\rangle$ and the orientifold plane and is determined entirely by the Möbius strip amplitude. In addition one finds some adjoint non-chiral matter in the A_{aa} open string sectors.

8. Examples

In this section we will exploit the formulas derived in the last sections for the construction of a number of explicit chiral models.

8.1 A simple current extension of the $(3)^5$ model

We start with the simplest Gepner model with levels $(3)^5$, corresponding to the quintic Calabi-Yau manifold. Since $(h_{21},h_{11})=(1,101)$, the pure A-type Gepner Model orientifold gives rise to 102 tadpole cancellation conditions for the 1984 Chan-Paton factors.

In order to reduce this to a treatable number we consider the extension of this model by the two simple currents

$$J_1=(0;2,-2,0,0,0;0,0,0,0,0)\,,\qquad J_2=(0;2,2,-4,0,0;0,0,0,0,0) \tag{8.1}$$

yielding the Hodge numbers $(h_{21},h_{11})=(49,5)$ and therefore leaving only six tadpole cancellation conditions. Note that the two simple currents in (8.1) are indeed relatively local.

Now we introduce the possible supersymmetric boundary states. It turns out that, after modding out the two simple currents and the Ω-action, we are left with 96 boundary states. These fall into three categories which can be described as follows. In the first class there are those 32 boundary states which are invariant under the Ω projection

$$|S^a_0;\prod_j(L^a_j,M^a_j,S^a_j)\rangle=|0;\prod_j(L^a_j,0,0)\rangle\,, \tag{8.2}$$

where the L^a_j denote any of the labels listed in appendix C. These states carry Chan-Paton indices N_{3i} with $i\in\{0,\ldots,31\}$. The second class contains the 32 states

$$|S^a_0;\prod_j(L^a_j,M^a_j,S^a_j)\rangle=|0;(L^a_1,-2,0)(L^a_2,0,0)(L^a_3,0,0)(L^a_4,2,0)(L^a_5,0,0)\rangle \tag{8.3}$$

with Chan-Paton indices N_{3i+1}. Finally, the third class are the states

$$|S_0^a; \prod_j (L_j^a, M_j^a, S_j^a)\rangle = |0; (L_1^a, -4, 0)(L_2^a, 0, 0)(L_3^a, 0, 0)(L_4^a, 4, 0)(L_5^a, 0, 0)\rangle \tag{8.4}$$

with Chan-Paton indices N_{3i+2}.

The next step is to evaluate the six tadpole cancellation conditions in terms of the 96 Chan-Paton factors. After a little algebra one can bring these conditions into a form with explicitly integer valued coefficients.

- Condition 1:

$$\begin{aligned} 12 = {} & 12N_0 + 2N_1 + 2N_2 + 6N_3 - 4N_4 + 6N_5 + 6N_6 - 4N_7 + 6N_8 + 18N_9 - 2N_{10} + \\ & +8N_{11} + 6N_{12} - 4N_{13} + 6N_{14} + 18N_{15} - 2N_{16} + 8N_{17} + 18N_{18} - 2N_{19} + \\ & +8N_{20} + 24N_{21} - 6N_{22} + 14N_{23} + 6N_{24} + N_{25} + N_{26} + 8N_{27} + 3N_{28} + 8N_{29} + \\ & +8N_{30} + 3N_{31} + 8N_{32} + 14N_{33} + 4N_{34} + 9N_{35} + 8N_{36} + 3N_{37} + 8N_{38} + \\ & +14N_{39} + 4N_{40} + 9N_{41} + 14N_{42} + 4N_{43} + 9N_{44} + 22N_{45} + 7N_{46} + 17N_{47} + \\ & +6N_{48} + N_{49} + N_{50} + 8N_{51} + 3N_{52} + 8N_{53} + 8N_{54} + 3N_{55} + 8N_{56} + 14N_{57} + \\ & +4N_{58} + 9N_{59} + 8N_{60} + 3N_{61} + 8N_{62} + 14N_{63} + 4N_{64} + 9N_{65} + 14N_{66} + \\ & +4N_{67} + 9N_{68} + 22N_{69} + 7N_{70} + 17N_{71} - 2N_{72} + 8N_{73} + 8N_{74} + 4N_{75} + \\ & +14N_{76} + 4N_{77} + 4N_{78} + 14N_{79} + 4N_{80} + 2N_{81} + 22N_{82} + 12N_{83} + 4N_{84} + \\ & +14N_{85} + 4N_{86} + 2N_{87} + 22N_{88} + 12N_{89} + 2N_{90} + 22N_{91} + 12N_{92} + \\ & +6N_{93} + 36N_{94} + 16N_{95}\,. \end{aligned} \tag{8.5}$$

- Condition 2:

$$\begin{aligned} 0 = {} & 5N_0 + 5N_1 + 5N_2 - 3N_3 - 3N_4 - 3N_5 - 3N_6 - 3N_7 - 3N_8 + 2N_9 + 2N_{10} + \\ & +2N_{11} - 3N_{12} - 3N_{13} - 3N_{14} + 2N_{15} + 2N_{16} + 2N_{17} + 2N_{18} + 2N_{19} + 2N_{20} - \\ & -N_{21} - N_{22} - N_{23} - 3N_{24} - 3N_{25} - 3N_{26} + 2N_{27} + 2N_{28} + 2N_{29} + 2N_{30} + \\ & +2N_{31} + 2N_{32} - N_{33} - N_{34} - N_{35} + 2N_{36} + 2N_{37} + 2N_{38} - N_{39} - N_{40} - N_{41} - \\ & -N_{42} - N_{43} - N_{44} + N_{45} + N_{46} + N_{47} - 3N_{48} - 3N_{49} - 3N_{50} + 2N_{51} + 2N_{52} + \\ & +2N_{53} + 2N_{54} + 2N_{55} + 2N_{56} - N_{57} - N_{58} - N_{59} + 2N_{60} + 2N_{61} + 2N_{62} - \\ & -N_{63} - N_{64} - N_{65} - N_{66} - N_{67} - N_{68} + N_{69} + N_{70} + N_{71} + 2N_{72} + 2N_{73} + \\ & +2N_{74} - N_{75} - N_{76} - N_{77} - N_{78} - N_{79} - N_{80} + N_{81} + N_{82} + N_{83} - N_{84} - \\ & -N_{85} - N_{86} + N_{87} + N_{88} + N_{89} + 2N_9 + N_{90} + N_{91} + N_{92}\,. \end{aligned} \tag{8.6}$$

- Condition 3:

$$\begin{aligned} 0 = {} & N_{25} - N_{26} - 2N_{27} + N_{28} - 2N_{30} + N_{31} - 2N_{33} + 2N_{34} - N_{35} - 2N_{36} + N_{37} - \\ & -2N_{39} + 2N_{40} - N_{41} - 2N_{42} + 2N_{43} - N_{44} - 4N_{45} + 3N_{46} - N_{47} - N_{49} + \\ & +N_{50} + 2N_{51} - N_{52} + 2N_{54} - N_{55} + 2N_{57} - 2N_{58} + N_{59} + 2N_{60} - N_{61} + \\ & +2N_{63} - 2N_{64} + N_{65} + 2N_{66} - 2N_{67} + N_{68} + 4N_{69} - 3N_{70} + N_{71}\,. \end{aligned} \tag{8.7}$$

- Condition 4:

$$\begin{aligned}0 = {} & 2N_{24} + N_{25} - 2N_{26} - 2N_{27} + N_{29} - 2N_{30} + N_{32} + N_{34} - N_{35} - 2N_{36} + N_{38} + \\ & + N_{40} - N_{41} + N_{43} - N_{44} - 2N_{45} + N_{46} - 2N_{48} - N_{49} + 2N_{50} + 2N_{51} - N_{53} + \\ & + 2N_{54} - N_{56} - N_{58} + N_{59} + 2N_{60} - N_{62} - N_{64} + N_{65} - N_{67} + N_{68} + \\ & + 2N_{69} - N_{70} \,. \end{aligned} \tag{8.8}$$

- Condition 5:

$$\begin{aligned}0 = {} & N_0 + 2N_1 + 3N_2 - N_3 - N_4 - 2N_5 - N_6 - N_7 - 2N_8 + N_{10} + N_{11} - N_{12} - \\ & - N_{13} - 2N_{14} + N_{16} + N_{17} + N_{19} + N_{20} - N_{21} - N_{23} - N_{24} - N_{25} - 2N_{26} + \\ & + N_{28} + N_{29} + N_{31} + N_{32} - N_{33} - N_{35} + N_{37} + N_{38} - N_{39} - N_{41} - \\ & - N_{42} - N_{44} - N_{45} + N_{46} - 3N_{48} - 2N_{49} + 2N_{51} + N_{52} + 2N_{54} + N_{55} - \\ & - N_{57} - N_{58} + 2N_{60} + N_{61} - N_{63} - N_{64} - N_{66} - N_{67} + N_{69} + 2N_{72} + N_{73} - \\ & - N_{75} - N_{76} - N_{78} - N_{79} + N_{81} - N_{84} - N_{85} + N_{87} + N_{90} - N_{94} \,. \end{aligned} \tag{8.9}$$

- Condition 6:

$$\begin{aligned}0 = {} & -N_1 + 2N_3 + N_5 + 2N_6 + N_8 + 2N_9 - N_{10} + N_{11} + 2N_{12} + N_{14} + 2N_{15} - \\ & - N_{16} + N_{17} + 2N_{18} - N_{19} + N_{20} + 4N_{21} - N_{22} + 2N_{23} + 2N_{24} - N_{25} + \\ & + 2N_{26} + 4N_{27} - 2N_{28} + N_{29} + 4N_{30} - 2N_{31} + N_{32} + 6N_{33} - 3N_{34} + 3N_{35} + \\ & + 4N_{36} - 2N_{37} + N_{38} + 6N_{39} - 3N_{40} + 3N_{41} + 6N_{42} - 3N_{43} + 3N_{44} + 10N_{45} - \\ & - 5N_{46} + 4N_{47} + 2N_{49} - 2N_{51} + 2N_{52} - 2N_{54} + 2N_{55} - 2N_{57} + 4N_{58} - 2N_{60} + \\ & + 2N_{61} - 2N_{63} + 4N_{64} - 2N_{66} + 4N_{67} - 4N_{69} + 6N_{70} + N_{73} + 2N_{76} + N_{77} + \\ & + 2N_{79} + N_{80} + 3N_{82} + N_{83} + 2N_{85} + N_{86} + 3N_{88} + N_{89} + 3N_{91} + N_{92} + \\ & + 5N_{94} + 2N_{95} \,. \end{aligned} \tag{8.10}$$

Clearly, it is not so easy to classify all possible solutions to these six equations.

Before we display at least a couple of non-trivial solutions, we would like to point out that the intersection numbers between pairs of the 96 boundary states do not always vanish. Therefore, in contrast to the B-type orientifold studied in [37], here chiral models are indeed possible. Moreover, since the intersection number does not always vanish, we can perform a highly non-trivial test of the entire presented formalism including the general sign factors in the Möbius strip amplitude. One can quite generally compute the non-abelian gauge anomaly on a stack of D-branes of type N_a in terms of all Chan-Paton indices. This is given by the following expression

$$\sum_{b\neq a} N_b \, (I_{a'b} - I_{ab}) + (N_a - 4)\tfrac{1}{2}\, (I_{a'a} + I_{oa}) + (N_a + 4)\tfrac{1}{2}\, (I_{a'a} - I_{oa}) \,. \tag{8.11}$$

Note that in the definition of the index I_{oa} the signs in the Möbius strip amplitude play a crucial role. Evaluating the 96 gauge anomalies (8.11) and inserting the 6 tadpole cancellation conditions, one can shown that they all indeed vanish. This constitutes a highly non-trivial test showing that everything is correct.

deg.	U(4) × U(2) × U(2) × U(2)
2	$(1,\overline{\mathbf{2}},\mathbf{2},1)$
1	$(1,\overline{\mathbf{2}},\overline{\mathbf{2}},1)$
2	$(1,\mathbf{2},1,\overline{\mathbf{2}})$
1	$(1,\mathbf{2},1,\mathbf{2})$
1	$(\mathbf{4},1,\mathbf{2},1)$
1	$(\overline{\mathbf{4}},1,1,\overline{\mathbf{2}})$
1	$(1,1,\overline{\mathbf{S}},1)$
1	$(1,1,1,\mathbf{S})$

Table 3: Massless chiral matter spectrum.

deg.	U(4) × U(2) × U(2) × U(2)
1	$(\mathbf{4},\overline{\mathbf{2}},1,1)+c.c.$
3	$(1,1,\mathbf{2},\overline{\mathbf{2}})+c.c.$
2	$(1,\mathbf{S},1,1)+c.c$
1	$(1,\mathbf{A},1,1)+c.c.$
5	$(\mathbf{Adj},1,1,1)$
5	$(1,\mathbf{Adj},1,1)$

Table 4: Massless non-chiral matter spectrum.

8.2 A chiral model

The choice $N_{10}=4$, $N_{17}=2$, $N_{25}=2$ and $N_{49}=2$ with the remaining Chan-Paton indices vanishing satisfies all six tadpole cancellation conditions. All four boundary states are not invariant under the Ω projection, so that we get a gauge group

$$G=\mathrm{U}(4)\times\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(2) \tag{8.12}$$

of rank $\mathrm{rk}(G)=10$. Computing the massless spectrum we find the chiral spectrum displayed in table 3.

Apparently, the non-abelian gauge anomaly cancels. This chiral massless spectrum is extended by the non-chiral one in table 4.

Note that the third and fourth stack of branes are rigid in the sense that there is no additional adjoint matter. This is one of the features which are very difficult to realize in intersecting brane worlds on toroidal orbifolds and which, of course, are really welcome in the string theoretical realization of the Standard Model as additional adjoint matter easily spoils asymptotic freedom and the nice gauge coupling unification properties of the low energy gauge groups [42].

If we extend the model further by the additional simple current

$$J_3=(0;2,2,2,-6,0;0,0,0,0,0) \tag{8.13}$$

we get the Greene-Plesser mirror of the quintic and this A-type model simply becomes the B-type model studied in [35, 36, 37].

8.3 A simple current extension of the $(1)^2\,(7)^3$ model

As a second non-trivial example we present a model derived from the $(1)^2\,(7)^3$ Gepner model. The Gepner model itself has Hodge numbers $(h_{21},h_{11})=(4,112)$ but leads after extending it by the two simple currents

$$J_1=(0;2,0,2,0,-8;0,0,0,0,0)\,,\qquad J_2=(0;-2,0,6,0,0;0,0,0,0,0) \tag{8.14}$$

to a model with Hodge numbers $(h_{21},h_{11})=(55,7)$.

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The possible supersymmetric boundary states come in two classes, where the first class contains 64 Ω-invariant states and the 64 states in the second class can be described as

$$|S_0^a;\prod_j(L_j^a,M_j^a,S_j^a)\rangle = |0;(L_1^a,-2,0)(L_2^a,0,0)(L_3^a,0,0)(L_4^a,0,0)(L_5^a,0,0)\rangle \tag{8.15}$$

with the labels L_j^a taken from the list in appendix C. Since we are more interested in unitary gauge groups in the following, we only consider these latter boundary states. Again after some algebra, the six independent tadpole cancellation conditions can be brought into a form with only integer coefficients and read:

- Condition 1:

$$\begin{aligned}20 = &\ 2N_0+2N_1+2N_2+2N_4+3N_5+4N_6+2N_7+2N_8+N_9+3N_{10}+N_{11}-\\ &-N_{13}+N_{14}+N_{15}+2N_{16}+6N_{17}+4N_{18}+2N_{19}+3N_{20}+9N_{21}+9N_{22}+\\ &+6N_{23}+N_{24}+6N_{25}+5N_{26}+4N_{27}-N_{28}+N_{30}+2N_{31}+2N_{32}+4N_{33}+\\ &+6N_{34}+4N_{35}+4N_{36}+9N_{37}+11N_{38}+7N_{39}+3N_{40}+5N_{41}+7N_{42}+\\ &+4N_{43}+N_{44}+N_{45}+N_{46}+2N_{49}+4N_{50}+4N_{51}+2N_{52}+6N_{53}+7N_{54}+\\ &+5N_{55}+N_{56}+4N_{57}+4N_{58}+3N_{59}+N_{60}+2N_{61}-N_{63}\,.\end{aligned} \tag{8.16}$$

- Condition 2:

$$\begin{aligned}0 = &\ 2N_0-N_1+N_2-N_3-N_4+N_7+2N_8-N_9+N_{10}-N_{11}-N_{16}+2N_{17}-\\ &-N_{18}-N_{21}+N_{22}+N_{23}-N_{24}+2N_{25}-N_{26}+N_{32}-N_{33}+N_{34}+N_{37}+\\ &+N_{40}-N_{41}+N_{42}-N_{48}+N_{51}+N_{52}+N_{53}-N_{55}-N_{56}+N_{59}\,.\end{aligned} \tag{8.17}$$

- Condition 3:

$$\begin{aligned}-2 = &\ -N_0+N_1-N_2-N_5-N_8+N_9-N_{10}+N_{16}-N_{17}-N_{19}-N_{20}-N_{21}-\\ &-N_{22}+N_{24}-N_{25}-N_{27}-N_{32}-N_{34}-N_{37}-N_{38}-N_{39}-N_{40}-N_{42}-\\ &-N_{49}-N_{54}-N_{55}-N_{57}\,.\end{aligned} \tag{8.18}$$

- Condition 4:

$$\begin{aligned}4 = &\ -2N_0+2N_1-N_2+N_3+N_4+N_6-2N_8+2N_9-N_{10}+N_{11}+2N_{16}-N_{17}+\\ &+2N_{18}+2N_{21}+N_{22}+N_{23}+2N_{24}-N_{25}+2N_{26}-N_{32}+2N_{33}+N_{35}+N_{36}+\\ &+N_{37}+2N_{38}+N_{39}-N_{40}+2N_{41}+N_{43}+N_{48}+N_{50}+N_{53}+\\ &+N_{54}+N_{55}+N_{56}+N_{58}\,.\end{aligned} \tag{8.19}$$

- Condition 5:

$$\begin{aligned}-4 = &\ -N_0+N_1-N_2+2N_4-2N_5+N_6-N_7+N_{12}-N_{13}+N_{14}+N_{16}-\\ &-N_{17}-N_{19}-2N_{20}+N_{21}-2N_{22}-N_{28}+N_{29}+N_{31}-N_{32}-N_{34}+N_{36}-\\ &-2N_{37}-N_{39}+N_{44}+N_{46}-N_{49}-N_{52}-N_{54}+N_{61}\,.\end{aligned} \tag{8.20}$$

- Condition 6:

$$
\begin{aligned}
-2 \;=\; & -2N_0+2N_1-N_2+N_3+4N_4-3N_5+3N_6-N_7+2N_{12}-2N_{13}+N_{14}- \\
& -N_{15}+2N_{16}-N_{17}+2N_{18}-3N_{20}+4N_{21}-N_{22}+2N_{23}-2N_{28}+N_{29}- \\
& -2N_{30}-N_{32}+2N_{33}+N_{35}+3N_{36}-N_{37}+3N_{38}+N_{44}-2N_{45}-N_{47}+ \\
& +N_{48}+N_{50}-N_{52}+2N_{53}+N_{55}-N_{60}-N_{62}\,. \qquad (8.21)
\end{aligned}
$$

Again one can check quite in general that as long as these six tadpole cancellation conditions are satisfied the non-abelian gauge anomalies do all cancel.

8.4 A chiral model

Choosing $N_{35}=4$, $N_{13}=2$, $N_{15}=2$ and $N_{19}=2$ with the remaining Chan-Paton indices vanishing satisfies all six tadpole cancellation conditions. The gauge group is the same as in the first example

$$G=\mathrm{U}(4)\times\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(2)\,, \qquad (8.22)$$

but as shown in table 5 the chiral massless spectrum is completely different.

As they should the non-abelian gauge anomalies cancel. This chiral massless spectrum is extended by the non-chiral one in table 6.

These two examples show that A-type orientifolds of Gepner models can lead to the very characteristics of the (supersymmetric) Standard model like unitary gauge groups of large enough rank, chirality, three generations and additional non-chiral (Higgs like) matter. The aim of this paper was solely to provide the necessary material for dealing with such models on the technical level. It is clear that a further systematic search has to be performed in order to really find models which come closer to the MSSM.

deg.	$\mathrm{U}(4)\times\mathrm{U}(2)\times\mathrm{U}(2)\times\mathrm{U}(2)$
2	$(\overline{\mathbf{4}},\overline{\mathbf{2}},1,1)$
2	$(\overline{\mathbf{4}},1,\overline{\mathbf{2}},1)$
4	$(\mathbf{4},1,1,\mathbf{2})$
2	$(1,\mathbf{2},\mathbf{2},1)$
3	$(1,\overline{\mathbf{2}},1,\overline{\mathbf{2}})$
1	$(1,1,\overline{\mathbf{2}},\overline{\mathbf{2}})$
3	$(\mathbf{A},1,1,1)$
2	$(1,\mathbf{S},1,1)$
1	$(1,\mathbf{A},1,1)$
1	$(1,1,\mathbf{S},1)$
1	$(1,1,1,\overline{\mathbf{S}})$
1	$(1,1,1,\mathbf{A})$

Table 5: Massless chiral matter spectrum.

9. Conclusions

In this paper we have investigated A-type orientifolds of Gepner models for their ability to give rise to some of the salient features of the supersymmetric Standard-Model like unitary gauge groups, chirality, family replication and large enough gauge groups. After having derived explicitly the general expressions for all relevant one-loop amplitudes, we have demonstrated by working out two examples in detail that all the rough Standard-Model features can be achieved by simple current extensions of Gepner models. This result is very promising and it would be very interesting to scan the whole plethora of such models for MSSM-like models. Given a concrete model, of course one would be interested in more refined data like Yukawa couplings or other pieces of the N=1 low energy effective action like the Kähler potential, gauge couplings and their one-loop threshold corrections.

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deg.	$\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{U}(2) \times \mathrm{U}(2)$
1	$(\mathbf{4}, \overline{\mathbf{2}}, 1, 1) + c.c.$
2	$(\mathbf{4}, 1, \overline{\mathbf{2}}, 1) + c.c.$
5	$(\mathbf{4}, 1, 1, \overline{\mathbf{2}}) + c.c.$
1	$(\mathbf{4}, 1, 1, \mathbf{2}) + c.c.$
1	$(1, \mathbf{2}, \overline{\mathbf{2}}, 1) + c.c.$
3	$(1, \mathbf{2}, \mathbf{2}, 1) + c.c.$
2	$(1, \mathbf{2}, 1, \overline{\mathbf{2}}) + c.c.$
2	$(1, 1, \mathbf{2}, \overline{\mathbf{2}}) + c.c.$
2	$(\mathbf{S}, 1, 1, 1) + c.c$
2	$(1, \mathbf{S}, 1, 1) + c.c.$
1	$(1, 1, \mathbf{S}, 1) + c.c.$
1	$(1, 1, 1, \mathbf{S}) + c.c.$
6	$(\mathbf{Adj}, 1, 1, 1)$
1	$(1, \mathbf{Adj}, 1, 1)$
1	$(1, 1, \mathbf{Adj}, 1)$
4	$(1, 1, 1, \mathbf{Adj})$

Table 6: Massless non-chiral matter spectrum.

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A. B-type Klein bottle projections

The B-type Klein bottle is computed from the diagonal partition function (3.20). Only those states coupling to one another in Z_D survive the Klein bottle Ω-projection which satisfy

$$\mu \cong \mu + b_0\beta_0 + b_1\beta_1 + \cdots + b_r\beta_r + \sum_\alpha 2\,\tau_\alpha j_\alpha\,, \tag{A.1}$$

where in addition the τ_α are constrained by the various δ-functions in (3.20). More abstractly, this is tantamount to requiring that the primaries be fixed points of the simple current algebra, i.e.

$$\phi^\lambda_\mu = \prod_{i=0}^{r} (J_i)^{b_i} \prod_{\alpha=1}^{I} J_\alpha^{2\,\tau_\alpha}\, \phi^\lambda_\mu\,. \tag{A.2}$$

To put it differently, the product of simple currents on the right-hand side of (A.2) acts as a stabilizer of ϕ^λ_μ. The set of non-trivial stabilizers in a RCFT is empty in the absence of short orbits, which clearly applies to our discussion of odd integer levels k_i. The only possibility is therefore that $\prod_{i=0}^{r}(J_i)^{b_i} \prod_{\alpha=1}^{I} J_\alpha^{2\,\tau_\alpha} = 1$. The case of non-vanishing exponents b_i, τ_α is easily seen to be excluded by the requirement stated above that none of the currents be generated by any combination of the others. We conclude that the fields appearing in the Klein bottle amplitude are precisely those for which the δ-functions in Z_D give non-vanishing contributions for $\tau_\alpha = 0$, i.e. those with integer monodromy $Q^{(\alpha)}_{\lambda,\mu}$. This additional projection of the possible states in the Klein bottle is the actual net effect of the simple current construction. Thus we find the simple-current extended B-type Klein bottle

$$K^B = 4\int_0^\infty \frac{dt}{t^3}\frac{1}{2^{r+1}}\frac{1}{\eta(2it)^2}\sum_{\lambda,\mu}^{\beta}\prod_{\alpha=1}^{I}\left(\delta^{(1)}(Q^{(\alpha)}_{\lambda,\mu})\right)\chi^\lambda_\mu(2it)\,. \tag{A.3}$$

The loop-channel amplitude is easily transformed into tree-channel by implementing the various δ- functions in terms of Lagrange multipliers, e.g.

$$\prod_{\alpha=1}^{I}\delta^{(1)}(Q^{(\alpha)}_{\lambda,\mu}) = \left(\prod_{\alpha=1}^{I}\frac{1}{\mathcal{N}_\alpha}\right)\sum_{\sigma_1=0}^{\mathcal{N}_1-1}\cdots\sum_{\sigma_I=0}^{\mathcal{N}_I-1}\exp\left(2\pi i\sum_j m_j\frac{\sum_\alpha\sigma_\alpha m_j^\alpha}{2k_j+4}\right). \tag{A.4}$$

Introducing P-matrices precisely as in [37], we finally arrive at the following form of the extended Klein bottle amplitude

$$\begin{aligned}\tilde{K}^B =& \frac{2^5\prod_j\sqrt{k_j+2}}{2^{\frac{3r}{2}}\,K\prod_\alpha\mathcal{N}_\alpha}\int_0^\infty dl\frac{1}{\eta(2il)^2}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{K-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1}\sum_{\sigma_1=0}^{\mathcal{N}_1-1}\cdots\sum_{\sigma_I=0}^{\mathcal{N}_I-1}(-1)^{\nu_0}\times\\ &\times\delta^{(4)}_{s_0',2+\nu_0+2\sum\nu_j}\prod_{j=1}^{r}\left(\frac{P^2_{l_j',\epsilon_j k_j}}{S_{l_j',0}}\delta^{(2k_j+4)}_{m_j',\nu_0+(1-\epsilon_j)(k_j+2)+\sum_\alpha\sigma_\alpha m_j^\alpha}\delta^{(4)}_{s_j',\nu_0+2\nu_j+2(1-\epsilon_j)}\right)\chi^{\lambda'}_{\mu'}(2il)\,.\end{aligned} \tag{A.5}$$

Consequently, the Ishibashi expansion of the crosscap states extracted from the Klein bottle is modified only by the analogous projections on integer monodromy:

$$\begin{aligned}|C\rangle_B =& \frac{1}{\kappa_c^B}\sum_{\lambda',\mu'}^{ev}\sum_{\nu_0=0}^{K-1}\sum_{\nu_1,\ldots,\nu_r=0}^{1}\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1}\sum_{\sigma_1=0}^{\mathcal{N}_1-1}\cdots\sum_{\sigma_I=0}^{\mathcal{N}_I-1}\Sigma(\lambda',\mu',\nu_0,\nu_j,\epsilon_j,\sigma_\alpha m_j^\alpha)\\ &\times\delta^{(4)}_{s_0',2+\nu_0+2\sum\nu_j}\prod_{j=1}^{r}\left(\frac{P_{l_j',\epsilon_j k_j}}{\sqrt{S_{l_j',0}}}\delta^{(2k_j+4)}_{m_j',\nu_0+(1-\epsilon_j)(k_j+2)+\sum_\alpha\sigma_\alpha m_j^\alpha}\delta^{(4)}_{s_j',\nu_0+2\nu_j+2(1-\epsilon_j)}\right)\times\\ &|\lambda',\mu'\rangle\rangle_c\,,\end{aligned} \tag{A.6}$$

where again we need to extract the correct sign from the consistent S-transformation of the Möbius-amplitude and the normalization factor is altered by the prefactors of the Lagrange multipliers as

$$\left(\frac{1}{\kappa_c^B}\right)^2 = \frac{2^5\prod_{j=1}^r\sqrt{k_j+2}}{2^{\frac{3r}{2}}\,K\prod_\alpha\mathcal{N}_\alpha}\,. \tag{A.7}$$

As is clear from the discussion of the A-type Klein bottle, the second δ-function above ensures that only those Ishibashi states contribute in $\tilde{K}^B$ which couple to their charge conjugate in Z_D.

B. P-transformation of the Möbius amplitude

Starting from

$$M_{\mu}^{\lambda} = \sum_{\nu_0=0}^{\frac{K}{2}} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\tau_1=0}^{\mathcal{N}_1} \cdots \sum_{\tau_{I'}=0}^{\mathcal{N}_{I'}} (-1)^{[h_{\mu}^{\lambda}(\nu_0,\nu_j,\tau_\alpha)-h_{\mu}^{\lambda}]} \hat{\chi}^{\lambda}_{\mu+2\nu_0\beta_0+\sum \nu_j\beta_j+\sum_{\alpha=1}^{I'} 2\tau_\alpha m_j^\alpha} \left(it+\frac{1}{2}\right), \tag{B.1}$$

we perform a P-transformation on the hatted characters by means of the P-matrices as given in [37].

Collecting the various contributions carefully, we find that we need to evaluate the sum

$$\sum_{\nu_0=0}^{\frac{K}{2}-1} e^{2\pi i\,\nu_0\left(1-\sum_\alpha 2\tau_\alpha Q^{\mathrm{U}(1)}(J_\alpha)-\frac{s_0'}{2}+\sum_j \frac{m_j'}{k_j+2}-\sum_j\frac{s_j'}{2}\right)} \sum_{\nu_1,\ldots,\nu_j}^{1} \left(\prod_{k<l}(-1)^{\nu_k\nu_l}\right) \times$$
$$\times(-1)^{\sum_l \nu_l\left(\frac{s_0+s_l}{2}+1-\frac{s_l'+s_0'}{2}+\epsilon_l\right)} \sum_{\tau_1=0}^{\mathcal{N}_1}\cdots\sum_{\tau_{I'}=0}^{\mathcal{N}_{I'}} \left(\prod_{\alpha<\beta}(-1)^{(2\tau_\alpha)\,Q_{\alpha\beta}(2\tau_\beta)}\right) e^{i\pi\sum_{\alpha=1}^{I'}\tau_\alpha x_\alpha} \tag{B.2}$$

where $Q^{\mathrm{U}(1)}(J_\alpha)$ denotes the U(1)-charge of J_α and

$$x_\alpha = h(J_\alpha) - Q^{(\alpha)}_{\lambda,\mu} + Q^{(\alpha)}_{\lambda',\mu'} + \sum_j \epsilon_j \frac{m_j^\alpha}{2}\,. \tag{B.3}$$

The sum over ν_0 yields the constraint

$$Q^{\mathrm{U}(1)}_{\lambda',\mu'} - 2\sum_\alpha \tau_\alpha Q^{\mathrm{U}(1)}(J_\alpha) + 1 = 0 \bmod 1\,, \tag{B.4}$$

which incorporates the Gepner projection on odd integer total U(1)-charge; the additional Gepner constraints are seen to be encoded in the various δ-functions appearing in the final expression.

The sum over ν_j is performed precisely as in [37], leading to

$$(-1)^s\, 2^{\frac{r+1}{2}} \prod_{k<l}(-1)^{\eta_k\eta_l}\, \delta^{(2)}_{\sum_j\eta_j,0} \tag{B.5}$$

with

$$\eta_j = \frac{s_0+s_j}{2} - \frac{s_0'+s_j'}{2} + \epsilon_j + 1\,. \tag{B.6}$$

As for the evaluation of the sum involving the τ_α, we note that due to mutual locality and in particular the factors of 2 appearing in front of the τ_α, the quadratic part equals 1, and we find immediately the desired constraint

$$Q^{(\alpha)}_{\lambda',\mu'} = 0 \bmod 1\,, \tag{B.7}$$

taking into account that all other terms in x_α are integral anyways. Collecting the other trivial factors from the P-matrices, we are lead to the result given in the text.

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C. L_j quantum numbers of the boundary states

This is the list of labels for the boundary states in the model derived from the $(3)^5$ Gepner model:

$$\begin{aligned}(L_1, L_2, L_3, L_4, L_5) \in \{&(0,0,0,0,0), (2,0,0,0,0), (0,2,0,0,0), (2,2,0,0,0), (0,0,2,0,0),\\ &(2,0,2,0,0), (0,2,2,0,0), (2,2,2,0,0), (0,0,0,2,0), (2,0,0,2,0),\\ &(0,2,0,2,0), (2,2,0,2,0), (0,0,2,2,0), (2,0,2,2,0), (0,2,2,2,0),\\ &(2,2,2,2,0), (0,0,0,0,2), (2,0,0,0,2), (0,2,0,0,2), (2,2,0,0,2),\\ &(0,0,2,0,2), (2,0,2,0,2), (0,2,2,0,2), (2,2,2,0,2), (0,0,0,2,2),\\ &(2,0,0,2,2), (0,2,0,2,2), (2,2,0,2,2), (0,0,2,2,2), (2,0,2,2,2),\\ &(0,2,2,2,2), (2,2,2,2,2)\} \,. \end{aligned} \tag{C.1}$$

And here we list the labels for the boundary states in the model derived from the $(1)^2\,(7)^3$ Gepner model:

$$\begin{aligned}(L_1, L_2, L_3, L_4, L_5) \in \{&(0,0,0,0,0), (0,0,2,0,0), (0,0,4,0,0), (0,0,6,0,0), (0,0,0,2,0),\\ &(0,0,2,2,0), (0,0,4,2,0), (0,0,6,2,0), (0,0,0,4,0), (0,0,2,4,0),\\ &(0,0,4,4,0), (0,0,6,4,0), (0,0,0,6,0), (0,0,2,6,0), (0,0,4,6,0),\\ &(0,0,6,6,0), (0,0,0,0,2), (0,0,2,0,2), (0,0,4,0,2), (0,0,6,0,2),\\ &(0,0,0,2,2), (0,0,2,2,2), (0,0,4,2,2), (0,0,6,2,2), (0,0,0,4,2),\\ &(0,0,2,4,2), (0,0,4,4,2), (0,0,6,4,2), (0,0,0,6,2), (0,0,2,6,2),\\ &(0,0,4,6,2), (0,0,6,6,2), (0,0,0,0,4), (0,0,2,0,4), (0,0,4,0,4),\\ &(0,0,6,0,4), (0,0,0,2,4), (0,0,2,2,4), (0,0,4,2,4), (0,0,6,2,4),\\ &(0,0,0,4,4), (0,0,2,4,4), (0,0,4,4,4), (0,0,6,4,4), (0,0,0,6,4),\\ &(0,0,2,6,4), (0,0,4,6,4), (0,0,6,6,4), (0,0,0,0,6), (0,0,2,0,6),\\ &(0,0,4,0,6), (0,0,6,0,6), (0,0,0,2,6), (0,0,2,2,6), (0,0,4,2,6),\\ &(0,0,6,2,6), (0,0,0,4,6), (0,0,2,4,6), (0,0,4,4,6), (0,0,6,4,6),\\ &(0,0,0,6,6), (0,0,2,6,6), (0,0,4,6,6), (0,0,6,6,6)\} \,. \end{aligned} \tag{C.2}$$

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